

MA210

Exercises 1

- (1) (a) There are 5 seats in a row and 12 people to choose from. How many different seatings are possible if every seat must be occupied by a person?  
(b) Now there are 12 seats in a row and 5 people to be seated. If every person must sit in a seat, how many different seatings are possible?
- (2) How many numbers between 2000 and 5000 can be formed from the digits 1,2,3,4,5,6 if repetition of digits is  
(a) allowed;  
(b) not allowed?
- (3) (a) How many 10-digit numbers can be formed by using each of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 exactly once?  
(b) And how many of these numbers are larger than 4,000,000,000?
- (4) A car registration is to consist of 3 letters followed by a number between 1 and 999. How many car registration numbers are possible?
- (5) You have an apple, a banana, a peach, a pear, and an apricot.  
(a) In how many ways can you distribute these pieces of fruit among 8 children if no child can receive more than one piece?  
(b) How many ways are there if any child can receive any number of pieces?
- (6) Suppose there are  $m$  girls and  $n$  boys in a class. What is the number of ways of arranging them in a line so that all the girls are together?
- (7) The names of the 12 months of the year are listed in a certain order. Given that June and July are not next to each other, how many such lists are possible?
- (8) Six people are to be seated around a circular table.  
(a) How many ways are there to achieve this?  
(b) And in how many ways can it be done if there are two people who refuse to sit next to each other?
- (9) An eight-person committee is to be formed from a group of 10 persons from London and 15 persons from outside the city.  
(a) In how many ways can the committee be chosen if the committee must contain 4 persons from each of the 2 groups?

- (b) In how many ways can the committee be chosen if the committee must contain more persons from outside London than from the city?
  - (c) In how many ways can the committee be chosen if the committee must contain at least 2 Londoners?
- (10) A certain game is played by 13 persons using a (standard) deck of 52 cards: there are four suits (Spades, Diamonds, Hearts, and Clubs) and 13 distinguishable cards (Ace, Two, ..., Ten, Jack, Queen, King) in each of the suits. A  $k$ -hand is a set of  $k$  different cards from this deck.
- (a) How many 4-hands are there?
- A player looks at her 4 cards and sees that she has only cards from one suit.
- (b) How many 4-hands that contain only cards from one suit are there?
- Another player looks at his 4 cards and sees that he has one cards from each suit.
- (c) How many 4-hands that contain one card from each suit exist?
- The player from (b) peeps at her left neighbour's cards and sees that this neighbour has only cards of one suit, but different from the one she has. Then she looks at her right neighbour's cards and, to her surprise, again sees only cards of one suit, and different from the suit she or the left neighbour has.
- (d) How many ways are there to divide 52 cards into thirteen 4-hands such that player 1 has cards of exactly one suit; player 2 has cards of exactly one suit, but not the same suit as player 1; player 3 has cards of exactly one suit, but not the same suit as player 1 or 2; and there are no restrictions for the hands of the other players?

You must justify the answers to all problems!

These exercises are to be handed in **before 4.55pm on January 19, 2009**.