

MA210

Exercises 2

- (1) The rules for the University of ABC five-a-side soccer competition specify that the members of each team must have birthdays in the same month. How many mathematics students are needed in order to guarantee that they can rise a team?
- (2)
 - (a) How many solutions are there of the equation $x_1 + x_2 + x_3 = 12$ with x_1, x_2, x_3 non-negative integers?
 - (b) How many solutions are there of the equation $x_1 + x_2 + x_3 = 12$ with x_1, x_2, x_3 positive integers?
- (3) A baseball team is made up of a pitcher and eight other players. The manager must choose the team from a group of 20 players, six of whom are pitchers. Note that non-pitchers cannot play at the pitcher's position.
 - (a) In how many ways can manager pick the team if the pitchers do not want to play in any other position than as a pitcher?
 - (b) In how many ways can the team be formed if the pitchers can also play as one of the other players?
- (4) Prove, by using a counting argument, that for $0 \leq k \leq r \leq n$,

$$\binom{n}{r} \cdot \binom{r}{k} = \binom{n}{k} \cdot \binom{n-k}{r-k}.$$

- (5) A domino has two numbers from $\{0, 1, 2, 3, 4, 5, 6\}$, one at each side of its (indistinguishable) ends. So, for example, there is a 3-3 domino, with a 3 at each end, and a 4-0 domino, which is the same as 0-4 domino. A set of dominoes contains one copy of each different domino. How many dominoes are there?
- (6) In how many ways can one choose 2 dominoes from the complete set so that at least one domino contains a 1 and at least one contains a 6?
- (7) What is the coefficient of $x^5 y^2 z^3$ in $(x + y + z)^{10}$?
- (8) How many different arrangements are there of the letters in the word MICROECONOMICS?
- (9) How many integers from 1 to 1000 are divisible by none of 5, 7, 11?

- (10) How many orderings are there of numbers $1, 2, \dots, 8$ in which none of the patterns 12, 34, 56, or 78 appears?
- (11) A (standard) deck consists of 52 cards: there are four suits (Spades, Diamonds, Hearts, and Clubs) and 13 distinguishable cards (Ace, Two, $\dots$, Ten, Jack, Queen, King) in each of the suits. A k -hand is a set of k different cards from this deck.
- (a) How many 13-hands are there?
 - (b) Use the Inclusion-Exclusion principle to find the number of 13-hands that contain at least one card from each suit.
 - (c) What is the chance that you obtain a 13-hand in which there are at most three suits?
- (12) The game of korfbal is played with teams of 6 players. Each team contains 3 female and 3 male players. A coach must select a korfbal team from 8 male and 7 female candidates.
- (a) How many choices are there to choose the team?
- In fact, the rules for forming a korfbal team of 6 players are a bit more complicated. A team actually consists of 3 pairs, each containing one female and one male player. One pair will play in the offense, one pair plays mid-field, and one pair plays defense. Our coach still has 8 male and 7 female candidates.
- (b) In how many ways can the coach choose 3 pairs (one offensive, one defensive and one for the mid-field), each pair consisting of one female and one male player?
- A coach of a different club has 7 female and 7 male players to form a korfbal team. These players are actually 7 married couples. The second coach knows from experience that a married couple should not form one of the pairs in a team.
- (c) Determine, using the Inclusion-Exclusion principle or otherwise, in how many ways the second coach can choose 3 pairs (one offensive, one defensive and one for the mid-field), each pair consisting of one female and one male player, so that none of the pairs is a married couple.

You must justify the answers to all problems!

These exercises are to be handed in **before 4.55pm on January 26, 2009.**