Discrete Mathematics   **Lent 2009**

MA210

Exercises 6

(1) Let $V = \{1, 2, \ldots, n\}$. How many different graphs with vertex set V are there?

(2) How many non-isomorphic graphs with four vertices are there? (Hint: the answer is not the same as the answer in Question 1 for $n = 4$.)

(3) Recall that the degree sequence of a graph is the list of all degrees of its vertices, written in non-increasing order. Prove that two isomorphic graphs must have the same degree sequence. Is it true that every two graphs with the same degree sequence are isomorphic? Justify your answer!

(4) A graph is 3-regular if all its vertices have degree 3. How many non-isomorphic 3-regular graphs with 6 vertices are there? And how many with 7 vertices?

(5) Determine which pairs of graphs below are isomorphic. Justify your answer!
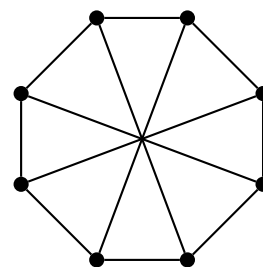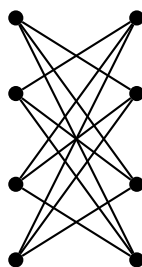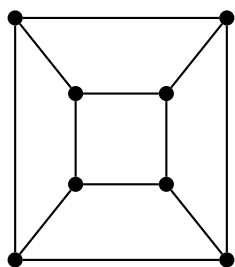
(6) Suppose that we have a graph with at least two vertices. Show that it is not possible that all vertices have different degrees.

(7) There are four married couples at a party. Various people shake hands, but of course no one shakes hands with his/her own wife or husband. At the end of the party, the host asks everybody else how many hands they shook and he receives seven different answers.
    How many hands did the wife of the host shake?

(8) Prove the following statements:
    (a) If there is a walk between two vertices x and y in some graph G, then there is also a path between x and y in G;
    (b) If G has a walk between vertices x and y and a walk between vertices y and z, then G also has a walk between x and z;
    (c) If G has a path between vertices x and y and a path between vertices y and z, then G also has a path between x and z.

You must justify the answers to all problems!

These exercises are to be handed in **before 13.55pm on February 24, 2009**.