Asset Pricing with Contrastive Adversarial Variational Bayes

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Abstract

Machine learning techniques have gained considerable attention in the field of empirical asset pricing. Conditioning on a broad set of firm characteristics, one of the most popular no-arbitrage workhorses is a nonlinear conditional asset pricing model that consists of two modules within a neural network structure, i.e., factor and beta estimates, for which we propose a novel contrastive adversarial variational Bayes (CAVB) framework. To exploit the factor structure, we employ adversarial variational Bayes that transforms the maximum-likelihood problem into a zero-sum game between a variational autoencoder (VAE) and a generative adversarial network (GAN), where an auxiliary discriminative network brings in arbitrary expressiveness to the inference model. To tackle the problem of learning indistinguishable feature representations in the beta network, we introduce a contrastive loss to learn distinctive hidden features of the factor loadings in correspondence to conditional quantiles of return distributions. CAVB establishes a robust relation between the cross-section of asset returns and the common latent factors with nonlinear factor loadings. Extensive experiments show that CAVB not only significantly outperforms prominent models in the existing literature in terms of out-of-sample total and predictive R^2s, but also delivers superior Sharpe ratios after transaction costs for both long-only and long-short portfolios.

1 Introduction

Factor models have become the workhorse for predicting asset returns using conditional information such as asset characteristics. In particular, dynamic factor models (DFM) are broadly employed. [Duan *et al.*, 2022] incorporate a variational autoencoder (VAE) into a DFM and propose a prior-posterior learning scheme for return prediction. [Wei *et al.*, 2023] design a hierarchical VAE-based DFM, which adaptively captures the regime-switching spatio-temporal relations in return prediction. [Xiang *et al.*, 2024] also emphasize the importance of regime switches in the DFM and rely on adversarial posterior factors to correct mapping deviations from prior factors. [Jia *et al.*, 2024] introduce adaptive graphs into a VAE-based factor model to capture dynamic asset relations. [Duan *et al.*, 2025] propose a hypergraph-based DFM with temporal contrastive learning that extracts additional hidden factors from residual information beyond the prior factors. [Shi *et al.*, 2025] design a surrogate model to predict fitness scores for factor mining and dynamically adjust factor weights in factor combinations. However, these papers ignore the well-known factor structure embedded in asset returns, which is the focus of this study.

A general factor model for empirical asset pricing assumes that the excess return $r_{i,t}$ of asset $i = 1, ..., N$ at time $t = 1, ..., T$ exhibits a K-factor structure as follows:

$$r_{i,t+1} = \beta'_{i,t} f_{t+1} + \epsilon_{i,t+1}, \tag{1}$$

where $\beta_{i,t} \in \mathbb{R}^{K\times 1}$ is the factor loading vector that can be interpreted as the exposures to K common latent factors $f_{t+1} \in \mathbb{R}^{K\times 1}$, and $\epsilon_{i,t+1}$ is the idiosyncratic error. The empirical estimation of (1) is challenging, as the factors are unobservable or unknown.

Most existing studies prespecify factors and estimate the corresponding betas via regression. A key limitation of this approach is its reliance on prior knowledge to identify relevant factors, a challenge often addressed in the literature through portfolio sorting based on characteristics. This results in the 'factor zoo' issue in empirical asset pricing [Cochrane, 2011].

[Kozak *et al.*, 2018] argue that given substantial commonality in the cross-section of returns, the absence of near-arbitrage opportunities implies that the stochastic discount factor (SDF) for explaining the return variations can be summarized by a few dominating factors. To address the issues of conventional principal component analysis applied to asset pricing, [Kelly *et al.*, 2019] propose the instrumented PCA (IPCA) method to estimate f_{t+1} as follows:

$$r_{i,t+1} = \beta(z_{i,t})' f_{t+1} + \epsilon_{i,t+1}. \tag{2}$$

Here $z_{i,t} \in \mathbb{R}^{P\times 1}$ is the observable characteristics of asset i with P strictly greater than K, and $\beta(z_{i,t})$ is a linear beta function of $z_{i,t}$. The dynamic factor loadings are given by $\beta(z_{i,t})' = z'_{i,t}\Gamma_\beta$, where $\Gamma_\beta \in \mathbb{R}^{P\times K}$ is a coefficient matrix. This setup specifies that characteristics are proxies for the sensitivities to common factors and thereby predict the average returns. Accordingly, [Kelly *et al.*, 2019] consider characteristics as instruments in PCA and find that the IPCA model with five factors significantly outperforms competing models published previously.

Using adaptive group LASSO, [Freyberger *et al.*, 2020] show that many of the previously identified characteristics do not offer incremental predictive information about expected returns. [Bryzgalova *et al.*, 2023a] employ Bayesian model averaging to handle the model specification problem with weakly identified factors. These studies still fall into the category of linear models. However, [Freyberger *et al.*, 2020] argue that the nonlinearities in characteristics are important for providing incremental information about the cross-section of expected returns. [Bryzgalova *et al.*, 2023b] propose a split-and-select model that spans the SDF based on decision trees, and show that it outperforms random forest or conventional deep learning methods.

[Gu *et al.*, 2021] also criticize the linearity assumption of the IPCA approach. They propose conditional autoencoder (CAE) asset pricing models that allow for a flexible nonlinear function of the covariates by applying a neural network method to learn $\beta(z_{i,t})$. They employ the autoencoder method to compress the N-dimensional r_{t+1} return matrix into a K-dimensional latent factor f_{t+1} using a neural network g, i.e., $f_{t+1} = g(r_{t+1})$. Then (2) is re-written in a nonlinear form as

$$r_{i,t+1} = \beta(z_{i,t})' g(r_{t+1}) + \epsilon_{i,t+1}. \tag{3}$$

Without an intercept, this equality respects the no-arbitrage condition. [Gu *et al.*, 2021] apply a fully connected network to estimate the model and find that characteristics predict average returns because they help to identify the risk exposures to common latent factors rather than capture mispricings.[1] They also show that IPCA is a special case of linear CAE when the covariance matrix of the characteristics is constant. Moreover, even if the covariance matrix varies over time, the empirical results remain similar.

With the rapid development of deep learning methods, recent studies have notably improved the performance of empirical asset pricing models by enhancing the learning capability of feature representation of the latent factors and factor loadings. Similarly to [Kozak *et al.*, 2020], who shrink redundant characteristics, [Chatigny *et al.*, 2021] rely on the attention mechanism [Vaswani *et al.*, 2017] to learn sparse features of a broad set of characteristics in an asset pricing model based on the high-dimensional optimization of the SDF weights. The resulting model significantly improves the performance of the baseline model without the attention mechanism.

Other research focuses on alternative training strategies. [Chen *et al.*, 2024] suggest that, within the SDF-based asset pricing framework, the loss function of weighted moments in the sample can be interpreted as weighted mean pricing errors. With this point of view, minimizing the objective loss is equivalent to imposing the no-arbitrage restriction. They employ adversarial learning to train neural networks, i.e., generalized adversarial networks (GANs), and achieve better performance than other classical deep learning frameworks, such as feed-forward or recurrent neural networks, including long short-term memory networks.

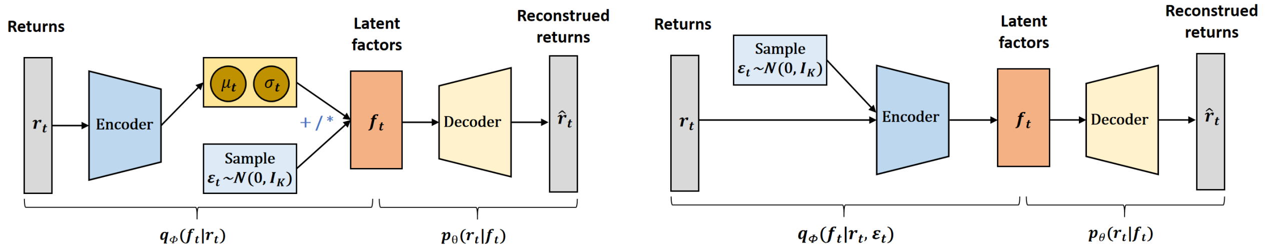
[Yang *et al.*, 2024] propose the conditional quantile variational autoencoder (CQVAE) network, which links the factor structure to the conditional quantiles of returns. Specifically, CQVAE learns the J quantile-dependent beta functions $\beta_j(z_t)$ via a multi-head neural network, where $j = 1, ..., J$ is the quantile index. Then it estimates $f(r_{t+1})$ via a VAE network. Compared to the standard autoencoder network, VAE alleviates the overfitting problem of deep learning methods. As a result, the CQVAE model significantly outperforms the CAE model.

Although the CQVAE model achieves impressive results, two major challenges remain. First, while it is economically meaningful for the CQVAE to learn quantile-dependent beta functions based on conditional return distributions, there is no empirical guarantee that the factor loadings learned by the beta network differ meaningfully across quantiles. This is because the preset quantile boundaries are not conditioned on additional information, making them suboptimal from a feature clustering perspective. If certain learned beta functions are indistinguishable from each other, the model will inevitably fail to explain the cross-section of asset returns. Second, as shown in the left panel of Figure 1, it is difficult for the approximate distributions of latent factors f_{t+1} learned by the VAE-based inference model $q_\phi(f_{t+1}|r_{t+1})$ in the CQVAE to capture the true posterior distributions, which limits the representational capability of latent factors. This is due to the limited expressiveness of the inference model, which constrains the performance of the resulting asset pricing model. Therefore, it is crucial to improve the feature extraction capacity of the factor network.

To address the above challenges, we propose a novel contrastive adversarial variational Bayes (CAVB) network to estimate (3). As shown in Figure 2, the proposed CAVB model consists of two neural network modules. The first module that improves the factor network aims to learn the common latent factors f_{t+1} by applying the adversarial variational Bayes (AVB) method. Especially, AVB improves the inference models $q_\phi(f_{t+1}|r_{t+1}, \epsilon_{t+1})$ by treating the noise ϵ_t as an additional input to the inference model, instead of adding it at the very end to construct the distributions in the VAE. This approach enables the inference network to learn the complex implicit probability distributions using adversarial training, which is achieved by introducing an auxiliary discriminative network as in the GAN that transforms the maximum likelihood problem into a zero-sum game where two networks contest with each other. This allows us to train the VAE with arbitrarily expressive inference models, and therefore, the representational capability of common latent factors is largely enhanced. The comparison of the conventional VAE and AVB is shown in Figure 1.

The second module of CAVB improves the beta network by introducing a contrastive learning method that focuses on extracting distinctive features for the return quantile-dependent beta functions $\beta_\tau(z_t)$. Given a set of characteristics z_t as inputs, each $\beta_\tau(z_t)$ is a neural network of three fully con-

[1] Accordingly, the alpha $\alpha_i = \mathbb{E}_t[r_{i,t+1}] - \mathbb{E}_t[\beta'_{i,t} f_{t+1}]$ can be tested against zero for mispricing.



(a) VAE (b) AVB

Figure 1: Comparison of the standard VAE and AVB architectures. The inference model $q_\phi(f_t|r_t)$ of the standard VAE is Gaussian, while the inference model $q_\phi(f_t|r_t, \epsilon_t)$ of the AVB can be arbitrary.

nected layers (FCLs) with a ReLu activation function and outputs the quantile-dependent factor loadings. To enhance the learning capacity of distinctive features between different quantile-dependent functions, we incorporate contrastive learning into the model training process. Contrastive learning was proposed by [He *et al.*, 2020] and aims to learn the feature representations of data by distinguishing similar data from dissimilar data through the use of appropriate positive and negative samples. To avoid learning indistinguishable feature representations for the beta functions, a contrastive loss is introduced by setting different quantile-dependent features as negative samples and those within the same conditional quantile distributions of returns as positive samples.

The remaining article is structured as follows. We present the overall architecture of the proposed CAVB model in Section 2. The architecture consists of a beta network and a factor network. In this section, we also provide the details of the training process of the CAVB network, including contrastive and adversarial learning methods. Section 3 presents empirical studies on a comprehensive real-world dataset. Section 4 concludes.

2 Methodology

We next present the overall architecture of the proposed framework, namely, the contrastive adversarial variational Bayes (CAVB), as described in Figure 2. It consists of two modules, namely a factor network using adversarial variational Bayes (AVB) and the beta network using conditional quantile-based contrastive learning. We also provide details of their training process.

2.1 Factor Network

In the factor network, we employ AVB, as proposed by [Mescheder *et al.*, 2017], wich unifies the VAE and GAN methods for learning the common latent factors f_t. The inference model of the conventional VAE method does not produce a sufficiently rich expressiveness to capture the true posterior distributions of the latent factors. To address this issue, AVB introduces an additional auxiliary discriminative network that allows the inference model to generate arbitrarily flexible and diverse probability distributions $q_\phi(f_t|r_t)$. The two competing networks, GAN and VAE, rephrase the maximum-likelihood problem as a zero-sum game that allows the model to closely approximate the true posterior distribution $p_\theta(f_t|r_t)$. In comparison with conventional VAE (see left panel of Figure 1), AVB (see right panel of Figure 1) treats noise as an additional input to the inference model, rather than adding it at the very end. This particular setup enables the inference network to learn complex implicit probability distributions via adversarial training.

The conventional VAE is trained by maximizing the evidence lower bound (ELBO) that estimates the marginal log-likelihood $\log p_\theta(r_t)$ in

$$\max_{\theta,\phi} \mathbb{E}_{f_t \sim q_\phi(f_t|r_t)} [\log p_\theta(r_t|f_t)] - \mathrm{KL}(q_\phi(f_t|r_t)||p_\theta(f_t)).$$

AVB shares the same objective function but is accompanied by an implicit inference model. The objective function of AVB is formally given by

$$\max_{\theta,\phi} \mathbb{E}_{r_t \sim p_{\mathcal{D}}(r_t)} \Big[\mathbb{E}_{f_t \sim q_\phi(f_t|r_t)}[\log p_\theta(r_t|f_t)] - \mathrm{KL}(q_\phi(f_t|r_t)||p_\theta(f_t))\Big], \tag{4}$$

where $p_{\mathcal{D}}(r_t)$ is the sample distribution of r_t. Since VAE has an explicit Gaussian inference model of $q_\phi(f_t|r_t)$ parameterized by a neural network, it is easy to optimize its objective function by an appropriate re-parameterization and stochastic gradient descent (SGD). However, as shown in the right panel of Figure 1, the inference model $q_\phi(f_t|r_t)$ of AVB is implicit. Hence, we cannot apply the re-parameterization to calculate the term $\mathrm{KL}(q_\phi(f_t|r_t)||p_\theta(f_t))$ in (4).

To solve this problem, we introduce a discriminative network $G_\psi(r_t, f_t)$ that takes the asset returns r_t and common latent factors f_t as inputs and gives corresponding discriminative values. The discriminative network first concatenates r_t and f_t, then feeds them into several linear layer blocks with residual connections, and finally outputs the discriminative values h_{out} as follows:

$$\begin{cases} h_{\text{in}} = W_{\text{in}}\,(r_t \copyright f_t) + b_{\text{in}}, \\ h = \mathrm{ELU}(W(h_{\text{in}}) + b + h_{\text{in}}), \\ h_{\text{out}} = W_{\text{out}}\,(h) + b_{\text{out}}, \end{cases}$$

where $W_{\text{in}} \in \mathbb{R}^{D\times(N+K)}$, $W \in \mathbb{R}^{D\times D}$, and $W_{\text{out}} \in \mathbb{R}^{1\times D}$ are the matrices of weight parameters, $b_{\text{in}} \in \mathbb{R}^D$, $b \in \mathbb{R}^D$, and $b_{\text{out}} \in \mathbb{R}^1$ are the bias parameters, D is the hidden dimension, and $\mathrm{ELU}(\cdot)$ is the exponential linear unit activation function.

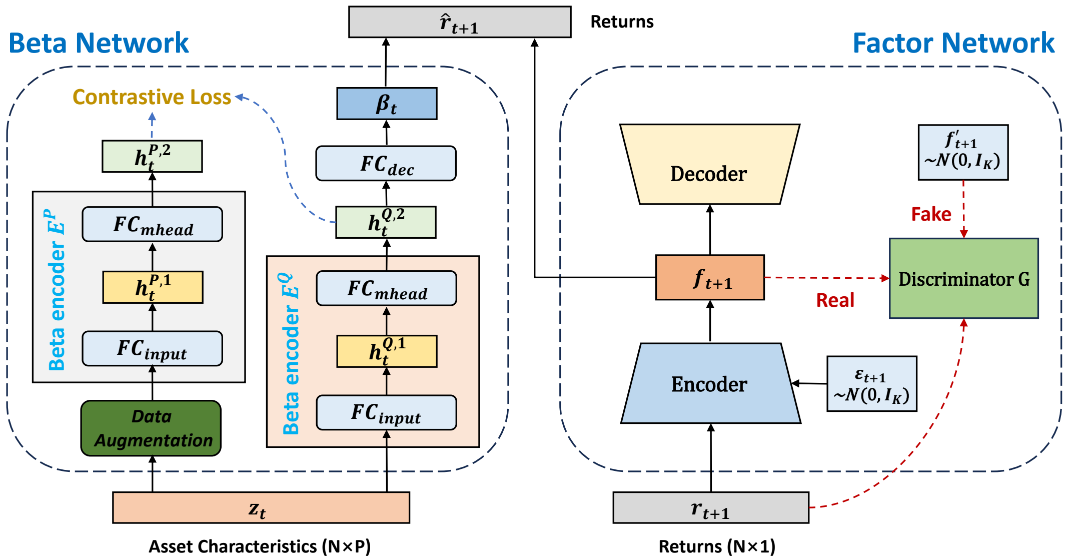


Figure 2: Architecture of the CAVB model. The CVAB consists of two modules, the beta network (on the left side) and the factor network (on the right side). The factor network compresses r_t into a common latent factor f_t by AVB. The beta network learns J different quantile-dependent factor loadings by a contrastive learning method. The cross-sectional expected returns $\hat{r}_t$ are then given by the product of the common latent factors and factor loadings.

The discriminative network in a standard GAN is used for judging whether the generated data are real or fake. In the proposed CAVB, we employ this type of discriminator to differentiate the pairs $(r_t,\ f_t)$ sampled from the posterior distributions $q_\phi(f_t|r_t)p_\mathcal{D}(r_t)$ and those sampled from the prior distributions $p(f_t)p_\mathcal{D}(r_t)$. To do so, we assign the following task to the discriminator $G_\psi(r_t, f_t)$:

$$\max_{\psi} \Big(\mathbb{E}_{r_t\sim p_\mathcal{D}(r_t)}\Big[\mathbb{E}_{f_t\sim q_\phi(f_t|r_t)}[\log(\sigma(G_\psi(r_t,f_t)))]\Big] + \mathbb{E}_{r_t\sim p_\mathcal{D}(r_t)}\Big[\mathbb{E}_{f_t\sim p(f_t)}\left[\log(1-\sigma(G_\psi(r_t,f_t)))\right]\Big]\Big), \tag{5}$$

where σ is the sigmoid function. When the discriminator maximizes the objective function in (4), in the literature the optimal discriminator commonly replaces $\mathrm{KL}(q_\phi(f_t|r_t)||p_\theta(f_t))$, and we can use

$$G_\psi(r_t, f_t)^* = \log q_\phi(f_t|r_t) - \log p_\theta(f_t).$$

As a result, we can replace $\mathrm{KL}(q_\phi(f_t|r_t)||p_\theta(f_t))$ by the optimal discriminator, and the objective function of AVB in (4) changes to

$$\max_{\theta,\phi} \mathbb{E}_{r_t\sim p_\mathcal{D}(r_t)}\Big[\mathbb{E}_{f_t\sim q_\phi(f_t|r_t)}[\log p_\theta(r_t|f_t) - G_\psi(r_t,f_t)^*]\Big]. \tag{6}$$

As shown in the right panel of Figure 1, AVB incorporates the noise $\epsilon_t \sim \mathcal{N}(0, I)$ as an additional input to the inference model. In this way, we can infer more flexible and diverse distributions with the proposed encoder. Using an appropriate re-parameterization, (6) can be rewritten as

$$\max_{\theta,\phi} \mathbb{E}_{r_t\sim p_\mathcal{D}(r_t)} \Big[\mathbb{E}_{\epsilon_t\sim\mathcal{N}(0,I)}[\log p_\theta(r_t|f_\phi^{\mathrm{enc}}(r_t,\epsilon_t)) - G_\psi(r_t, f_\phi^{\mathrm{enc}}(r_t,\epsilon_t))^*]\Big], \tag{7}$$

where $f_\phi^{\mathrm{enc}}(r_t,\epsilon_t) = q_\phi(f_t|r_t,\epsilon_t)$ is the encoder of AVB. Next, we apply Monte Carlo to estimate the first term of the ELBO in (7) through sampling the data M times, similar to M min-batch training:

$$\mathbb{E}_{r_t\sim p_\mathcal{D}(r_t)}\Big[\mathbb{E}_{\epsilon_t\sim\mathcal{N}(0,I)}[\log p_\theta(r_t|f_\phi^{\mathrm{enc}}(r_t,\epsilon_t))]\Big] \approx \frac{1}{M}\sum_{m=1}^{M}\mathbb{E}_{\epsilon_t\sim\mathcal{N}(0,I)}[\log p_\theta(r_t|f_\phi^{\mathrm{enc}}(r_t,\epsilon_t))]. \tag{8}$$

The objective function in (7) has two competing terms. To optimize their difference, we resort to an alternate training scheme, namely an adversarial learning method, see Subsection 2.3.

We provide more details on the network structures of the encoder and decoder in Online Appendix A.1.

2.2 Beta Network

The beta network is also an encoder-decoder neural network architecture that aims to learn J different quantile-dependent factor loadings for asset pricing. Specially, we construct J different beta functions $\beta_\tau(z_{i,t})$ using a multi-head structure. The encoder with a multi-head structure is used for extracting the hidden features H_t^1 in different conditional quantile distributions of returns. With the hidden features H_t^1 as inputs, the decoder constructs the quantile-dependent factor loadings $\beta_{\tau,t}$ for each of the quantiles. We delegate the network structures of the encoder and decoder to Online Appendix A.2.

Contrastive learning is an ideal tool to enhance the differentiations in extracted features between different quantile-dependent beta functions. The first step of contrastive learning is to construct positive and negative pairs. In the cross-sectional asset pricing exercise, it is economically intuitive to treat the hidden features from the same quantile as the positive pairs and the hidden features from different quantiles as the negative pairs. Moreover, we construct two beta encoders with the same architecture but different parameters, i.e., encoder P and encoder Q to capture the hidden features. As one of the most popular ways of data augmentation, given the inputs z_t at each point of time, we employ the encoder Q to extract the hidden features $h_t^{Q,2}$. Then we add random Gaussian noise into the asset characteristics z_t to obtain new inputs z'_t, and feed z'_t into the encoder P yielding another set of hidden features $h_t^{P,2}$. Finally, given these hidden features, we compute the contrastive loss at time t as

$$\mathcal{L}_{\text{cl}} = \frac{1}{J}\sum_{j=1}^{J} \log \frac{\exp(h_{j,t}^{Q,2} * h_{j,t}^{P,2})}{\sum_{m=1}^{J} \exp(h_{m,t}^{Q,2} * h_{m,t}^{P,2})}. \tag{9}$$

We update the beta network by SGD with contrastive loss. Encoder Q is used for the entire empirical analysis while encoder P is only used in the training process.

2.3 Training in the CAVB Network

We train the factor network and the beta network in two steps. First, we train the factor network using AVB with its objective function in (7). To this end, we use SGD to update the parameter ϕ in the encoder and decoder in the factor network by using the loss function

$$\mathcal{L}_{\text{factor}} = \frac{1}{M}\sum_{m=1}^{M}\Big((r_t - f_\theta^{\text{dec}}(f_t))^2 + G_\psi(r_t, f_\phi^{\text{enc}}(r_t,\epsilon_t))\Big), \tag{10}$$

where $f_\theta^{\text{dec}}(f_t)$ is the decoder network, and M and $G_\psi(r_t, f_\phi^{\text{enc}}(r_t,\epsilon_t))$ are as in Subsection 2.1.

As we rely on adversarial training in the factor network, we also need to train the discriminator (i.e., the parameter ψ). According to the corresponding objective function in (5), the loss function is given by

$$\mathcal{L}_G = -\log(\sigma(G_\psi(r_t, f_t))) - \log(1-\sigma(G_\psi(r_t, f_t))), \tag{11}$$

where σ is the sigmoid function. Therefore, to optimize the encoder, the decoder, and the discriminator altogether, we arrange an alternate training scheme based on adversarial learning for the factor network. The overall training process of the factor network is introduced in Algorithm 1 of Online Appendix B. It yields the estimated parameters of the factor network, i.e., $\hat{\theta}$, $\hat{\phi}$, and $\hat{\psi}$. Then the common latent factors $\hat{f}_{t+1}$ can be constructed by the estimated encoder.

Given the estimated latent factor $\hat{f}_{t+1}$ and z_t, we can train the beta network in the second step using SGD with the loss

$$\mathcal{L}_{\text{beta}} = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{1}{NJ}\sum_{i=1}^{N}\sum_{j=1}^{J}\rho_{\tau_j}(r_{i,t+1} - \beta_{\tau_j}(z_{i,t})'\hat{f}_{t+1}) + \mathcal{L}_{\text{cl}}\right), \tag{12}$$

where $\rho_{\tau_j}(u) = |u|(\tau_j \mathbf{1}_{u\geq 0} + (1-\tau_j)\mathbf{1}_{u<0})$ denotes the check function with τ_j as the quantile [Koenker and Bassett Jr, 1978] and $\mathcal{L}_{\text{cl}}$ is the contrastive loss of (9). Algorithm 2 in Online Appendix C shows the min-batch training process of the beta network. Following Algorithm 1, we obtain the estimated parameters of the beta network $\hat{\psi}$, and accordingly the estimated quantile-dependent beta functions $\hat{\beta}_\tau(z_t)$. Given $\hat{\beta}_\tau(z_t)$ and the estimated latent factor $\hat{f}_{t+1}$, we can compute the quantiles of returns $\hat{Q}_{i,t}(\tau_j) = \hat{\beta}_\tau(z_t)'\hat{f}_{t+1}$. Finally, we plug $\hat{Q}_{i,t}(\tau_j)$ into the discrete conditional distribution function of returns and calculate the cross-section of expected returns $\hat{r}_{i,t}$ from the asset pricing model.

The above training process of the CAVB model is divided into two steps. Alternatively, one might simultaneously optimize the factor network and the beta network within one step. The corresponding loss function is then given by

$$\mathcal{L} = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{1}{NJ}\sum_{i=1}^{N}\sum_{j=1}^{J}\rho_{\tau_j}(r_{i,t+1} - \beta_{\tau_j}(z_{i,t})' f_\theta^{\text{dec}}(f_{t+1})) + \mathcal{L}_{\text{cl}}) + G_\psi(r_t, f_\phi^{\text{enc}}(r_t,\epsilon_t))\right). \tag{13}$$

This one-step approach aims to price and predict asset returns by adaptively learning latent factors and factor loadings, optimizing the parameters of the beta network δ, the encoder θ, and the decoder ϕ, simultaneously. The algorithm is shown in Algorithm 3 in Online Appendix D.

3 Experiment

3.1 Dataset

We evaluate the proposed CAVB model using the Open Source Asset Pricing dataset consisting of a variety of monthly firm characteristics. We focus on liquidly traded stocks to ensure the scalability of portfolio strategies, and remove the binary characteristics. All in all, the dataset consists of 96 different observable firm characteristics and individual returns (obtained from the CRSP database) of stocks traded on NYSE, AMEX, and NASDAQ from January 1980 to December 2022. Missing values in the dataset are replaced by the corresponding cross-sectional medians as in [Gu *et al.*, 2021]. To capture the time-varying market states, we adopt a moving window of 31 years, split by a 20-year training sample, 10-year validation sample, and 1-year test sample, starting from January 1980. Therefore, the moving window rolls 13 times up to December 2022, yielding an out-of-sample period of 13 years to test the empirical asset pricing models.

3.2 Baselines

In the empirical analysis, we compare CAVB with the latest asset pricing models, including:

- Instrumented PCA (**IPCA**) [Kelly *et al.*, 2019] use firm characteristics as instruments for learning factor loadings in PCA to establish relations between average returns and latent factors.
- Conditional Autoencoder (**CAE**) [Gu *et al.*, 2021] employ the autoencoder method to learn the nonlinear relations between firm characteristics and average returns in a neural network-based asset pricing model.
- Attention-Guided Deep Learning (**AGDL**) [Chatigny *et al.*, 2021] propose an attention-guided deep learning

Model	Test Assets	Total R^2 (%)						Predictive R^2 (%)					
		$K=1$	$K=2$	$K=3$	$K=4$	$K=5$	$K=6$	$K=1$	$K=2$	$K=3$	$K=4$	$K=5$	$K=6$
IPCA	$\boldsymbol{r}_t$	8.12	13.26	18.54	20.76	22.43	23.11	3.06	3.52	6.85	7.85	8.22	8.56
	$\boldsymbol{x}_t$	9.57	15.49	21.85	24.47	26.35	27.23	3.62	4.13	8.01	9.22	9.56	10.06
CAE	$\boldsymbol{r}_t$	15.27	17.62	18.43	19.87	21.34	22.75	2.13	2.45	2.62	2.71	2.74	2.85
	$\boldsymbol{x}_t$	18.06	20.85	21.78	23.45	25.21	26.89	2.53	2.89	3.12	3.22	3.27	3.39
AGDL	$\boldsymbol{r}_t$	21.07	23.82	24.76	26.33	28.13	29.56	1.84	1.95	2.11	2.09	2.14	2.21
	$\boldsymbol{x}_t$	24.84	28.10	29.19	31.20	33.18	34.85	2.19	2.34	2.51	2.48	2.54	2.63
GAN	$\boldsymbol{r}_t$	22.18	24.32	25.81	25.73	26.89	27.94	1.93	1.98	2.09	2.03	2.11	2.14
	$\boldsymbol{x}_t$	26.17	28.72	30.45	30.34	32.04	32.97	2.27	2.33	2.45	2.39	2.48	2.52
CQVAE	$\boldsymbol{r}_t$	38.38	40.26	41.13	41.60	41.63	41.46	3.81	5.62	7.43	8.27	9.18	10.85
	$\boldsymbol{x}_t$	43.31	45.43	46.38	46.90	46.98	46.79	4.27	6.29	8.32	9.27	10.28	12.16
CAVB	$\boldsymbol{r}_t$	40.98	41.35	41.77	42.03	42.54	42.65	6.81	8.98	10.46	12.37	15.17	16.09
	$\boldsymbol{x}_t$	47.53	48.03	48.51	48.71	49.39	49.54	8.07	10.62	12.52	14.76	18.07	19.15
CAVB$_{\text{w/o A}}$	$\boldsymbol{r}_t$	39.94	40.85	41.94	41.98	42.12	42.17	5.12	6.38	8.12	9.64	10.77	11.14
	$\boldsymbol{x}_t$	46.73	47.79	48.62	48.69	49.27	49.34	5.65	8.40	8.97	10.64	11.88	12.51
CAVB$_{\text{w/o C}}$	$\boldsymbol{r}_t$	40.28	40.92	41.36	41.84	42.23	42.02	5.83	7.61	8.27	11.94	13.37	14.06
	$\boldsymbol{x}_t$	45.56	46.28	46.78	47.32	47.74	47.53	6.47	8.41	9.17	13.17	14.74	15.51
CAVB$_{\text{1-Step}}$	$\boldsymbol{r}_t$	**51.90**	**53.31**	**52.68**	**51.90**	**53.64**	**53.76**	**7.12**	**10.73**	**10.64**	**13.16**	**18.51**	**19.99**
	$\boldsymbol{x}_t$	**60.82**	**60.98**	**61.84**	**60.94**	**62.74**	**63.77**	**8.58**	**11.33**	**12.46**	**14.92**	**20.30**	**20.96**

Table 1: The out-of-sample total R^2 and predictive R^2 comparisons of all competing models with different numbers of latent factors K. The figures in bold (underlined) indicate the best (second best) results.

method to learn the sparse features of firm characteristics in a weighted SDF-based asset pricing model.

- Generative Adversarial Networks (**GAN**) [Chen *et al.*, 2024] apply adversarial learning method to train nonlinear neural networks with an objective function of weighted sample moments, which also guarantee the absence of arbitrage.
- Conditional Quantile Variational Autoencoder (**CQVAE**) [Yang *et al.*, 2024] employ the VAE method to learn the conditional quantile-based relations between average returns and firm characteristics in terms of factor loadings.

The two main CAVB models are two-step **CAVB** and one-step **CAVB$_{\text{1-Step}}$**. To validate the effectiveness of contrastive learning and AVB, we show two variants of CAVB in the ablation study: **CAVB$_{\text{w/o A}}$** uses plain vanilla VAE instead of the AVB method; **CAVB$_{\text{w/o C}}$** trains the quantile-dependent beta network without the use of contrastive learning.

3.3 Performance Metrics

Following [Kelly *et al.*, 2019], [Gu *et al.*, 2021], and [Yang *et al.*, 2024], we evaluate the out-of-sample performance of all competing asset pricing models by two metrics in the test data. The first one is the total R^2 defined as

$$R^2_{\text{total}} = 1 - \frac{\sum_{i,t}(r_{i,t+1} - \hat{r}^{\text{total}}_{i,t+1})^2}{\sum_{i,t} r^2_{i,t+1}},$$

where

$$\hat{r}^{\text{total}}_{i,t+1} = \sum_{j=1}^{J} (\tau^*_j - \tau^*_{j-1})\hat{\beta}_{\tau_j}(z_{i,t})'\hat{f}_{t+1},$$

$\hat{\beta}_{\tau_j}(z_{i,t})$ is the estimated quantile-dependent beta function, and $\hat{f}_{t+1}$ the estimated latent factors. The total R^2 measures how well a model explains the cross-sectional variations of asset returns or the characteristic-managed portfolio.

The second evaluation metric is the predictive R^2 defined as R^2_{total} but with $\hat{r}^{\text{total}}_{i,t+1}$ replaced by

$$\hat{r}^{\text{pred}}_{i,t+1} = \sum_{j=1}^{J} (\tau^*_j - \tau^*_{j-1})\hat{\beta}_{\tau_j}(z_{i,t})'\hat{\lambda}_t,$$

where $\hat{\lambda}_t = \mathbb{E}_t[f_{t+1}]$ measures the expected risk compensation, computed as the sample average of $\hat{f}$ up to time t. We use a standard 60-month rolling window to calculate $\hat{\lambda}_t$.[2] In addition to testing on individual stock returns r_{t+1}, we also test on characteristic-managed portfolios $x_{t+1} = (z_t' z_t)^{-1} z_t' r_{t+1}$, where z_t is an $N \times P$ weighting matrix that constructs P characteristic-based portfolios from N assets.

3.4 Statistical Evaluation

Table 1 reports the out-of-sample total R^2 and predictive R^2 of the CAVB models and the baseline models with $K = 1, 2, \ldots, 6$ latent factors as in [Kelly *et al.*, 2019], [Gu *et al.*, 2021], and [Yang *et al.*, 2024], for individual stock returns $r_{i,t}$ and characteristic-managed portfolios x_t. Similar to these three papers, both the total R^2 and predictive R^2 generally increase with the number of the latent factors but the performance seems to reach its peak around $K = 5$ or $K = 6$. A smaller K results in the loss of valuable information, while a larger K leads to model overfitting; see also [Kelly *et al.*, 2019] for a justification to test up to 6 factors.

As we can see in Table 1, compared with the competing models, the proposed (one-step or two-step) CAVB models achieve the best performance in terms of both total and predictive R^2. The two-step **CAVB** follows the standard setup of [Gu *et al.*, 2021] and [Yang *et al.*, 2024] to estimate the factor and beta networks separately, while the one-step **CAVB$_{\text{1-Step}}$** focuses on the asset returns by adaptively learning and simultaneously optimizing the parameters of both networks. For

[2] We find that the empirical results are not sensitive to the rolling window size in the dataset.

Model	Long-Only Portfolios						Long-Short Portfolios					
	$K=1$	$K=2$	$K=3$	$K=4$	$K=5$	$K=6$	$K=1$	$K=2$	$K=3$	$K=4$	$K=5$	$K=6$
IPCA	0.76	0.87	1.38	2.21	2.94	3.41	0.79	0.94	1.53	2.46	2.98	3.74
CAE	0.27	0.53	0.67	0.83	1.02	1.11	0.38	0.48	0.63	0.79	0.93	0.91
AGDL	0.47	0.68	1.21	1.67	1.92	2.20	0.52	0.78	1.37	1.93	2.39	2.56
GAN	0.59	0.87	1.41	1.89	2.28	2.39	0.67	0.92	1.74	2.10	2.34	2.64
CQVAE	0.68	0.92	1.48	1.83	2.17	2.24	0.71	0.98	1.52	1.71	1.78	1.86
CAVB	0.82	<u>1.19</u>	**2.28**	**3.14**	**3.96**	**4.42**	**0.94**	**1.36**	**2.47**	<u>3.23</u>	**4.32**	**4.62**
$\mathrm{CAVB}_{\mathrm{w/o\ A}}$	0.78	0.96	1.53	2.48	<u>3.15</u>	<u>3.63</u>	<u>0.91</u>	0.89	1.64	2.57	<u>3.42</u>	3.38
$\mathrm{CAVB}_{\mathrm{w/o\ C}}$	<u>0.84</u>	1.04	1.98	2.53	2.81	3.46	0.87	1.22	<u>1.87</u>	2.48	3.02	<u>3.86</u>
$\mathrm{CAVB}_{\mathrm{1\text{-}Step}}$	**1.25**	**1.41**	<u>2.04</u>	<u>2.85</u>	2.54	3.17	0.88	<u>1.27</u>	1.80	**3.32**	2.47	3.53

Table 2: The out-of-sample Sharpe ratios of long-only and long-short portfolios with 30 bps transaction costs in comparison with different numbers of latent factors K. The figures in bold (underlined) indicate the best (second best) results.

the two-step models, its variant $\mathbf{CAVB}_{\mathbf{w/o\ A}}$ performs better in the total R^2 when $K = 3$. This result may be driven by the fact that we did not fine-tune the hyperparameters in the two-step optimization. Both CAVB variants outperform the competing models. In particular, the $\mathbf{CAVB}_{\mathbf{1\text{-}Step}}$ and CAVB models considerably outperform the competing models by large margins in the predictive R^2, e.g., the $\mathbf{CAVB}_{\mathbf{1\text{-}Step}}$ (**CAVB**) improves the predictive R^2 of **CQVAE** by 84.24% (48.29%) for individual stocks and by 72.37% (57.48%) for characteristic-managed portfolios when $K = 6$. These findings indicate that the CAVB model possesses not only better explanation power in the cross-sectional return variations but also far more robust predictive power for future returns.

Moreover, to evaluate the components of the proposed CAVB model, we investigate its two variants, i.e., $\mathbf{CAVB}_{\mathbf{w/o\ A}}$ and $\mathbf{CAVB}_{\mathbf{w/o\ C}}$, to verify the importance of using AVB to learn the latent factor and applying contrastive learning to train the quantile-dependent beta network. As Table 1 illustrates, both components contribute to the superior performance of the CAVB model. Specifically, $\mathbf{CAVB}_{\mathbf{w/o\ C}}$ consistently outperforms **CQVAE** across all cases in terms of evaluation metrics, numbers of latent factors, and tested assets. This indicates that AVB can capture common latent factors with higher-quality features. Similarly, the performance of $\mathbf{CAVB}_{\mathbf{w/o\ A}}$ also demonstrates the usefulness of contrastive learning in learning distinctive features of the quantile-dependent factor loadings across different conditional quantile distributions of returns, although its contribution is not as significant as that of AVB.

3.5 Economic Evaluation

To evaluate the economic contribution of the CAVB models, we conduct an out-of-sample portfolio trading experiment according to the rank of the predicted returns of the individual assets and run horse races for all competing models with different numbers of latent factors. Specifically, we sort the predictive returns $\hat{r}^{\text{pred}}_{i,t+1}$ by different models and select stocks within the top 10% and bottom 10% of predicted returns. To construct the long-only portfolios, we buy and hold the top 10% stocks and rebalance the portfolio monthly. To construct the long-short zero-investment portfolios, we short-sell the bottom 10% of stocks by short-selling, rebalancing them also at a monthly frequency. We make sure that the funding leg has the same portfolio size as the investment leg. All selected stocks are equally weighted in the portfolios, and the transaction cost for portfolio rebalancing is assumed to be 30 basis points (bps), which is reasonably high for U.S. stocks. We evaluate the economic performance via Sharpe ratios of the constructed portfolios on the test data.

Table 2 reports the out-of-sample Sharpe ratios of both long-only and long-short portfolios for all competing models with different numbers of latent factors K. We find that the proposed CAVB models consistently and significantly outperform the competing models in terms of Sharpe ratio. As expected, the long-short portfolios overall perform better than the long-only ones. This is due to the benefit of portfolio hedging. Surprisingly, the linear conditional asset pricing model **IPCA** is more effective in converting its statistical predictive power into economic value than other non-CAVB models in terms of Sharpe ratio. Yet, **CAVB** manages to achieve higher Sharpe ratios than **IPCA**, namely 29.62% higher for long-only portfolios and 23.53% higher for long-short portfolios with $K = 6$. **CAVB** also consistently and significantly outperforms its variants, especially when the number of latent factors gets larger, except for the case $K = 1$ for long-only portfolios but the difference in Sharpe ratios is minimal. In terms of Sharpe ratio, it seems that the AVB (contrastive learning) module is more effective in converting its statistical predictive power into economic value when K is smaller (bigger). These findings suggest that both CAVB components, i.e., the AVB and contrastive learning methods, are equally important in profit generation.

4 Conclusion

We have presented a novel contrastive adversarial variational Bayes (CAVB) network for nonlinear conditional asset pricing with a broad set of firm characteristics. It establishes a robust connection between the conditional quantile distributions of returns and the latent factor structure with nonlinear factor loadings. Extensive experiments show that proposed CAVB model significantly outperforms established models in terms of out-of-sample total and predictive R^2s. When applied to portfolio trading, it delivers superior transaction-cost adjusted Sharpe ratios for both long-only and long-short portfolios in comparison to the competing models.

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